

Muon capture on light nuclei

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Abstract This work investigates the muon capture reactions ${}^2\text{H}(\mu^-, \nu_\mu)nn$ and ${}^3\text{He}(\mu^-, \nu_\mu){}^3\text{H}$ and the contribution to their total capture rates arising from the axial two-body currents obtained imposing the partially-conserved-axial-current (PCAC) hypothesis. The initial and final $A = 2$ and 3 nuclear wave functions are obtained from the Argonne v_{18} two-nucleon potential, in combination with the Urbana IX three-nucleon potential in the case of $A = 3$. The weak current consists of vector and axial components derived in chiral effective field theory. The low-energy constant entering the vector (axial) component is determined by reproducing the isovector combination of the trinucleon magnetic moment (Gamow-Teller matrix element of tritium beta-decay). The total capture rates are $393.1(8) \text{ s}^{-1}$ for $A = 2$ and $1488(9) \text{ s}^{-1}$ for $A = 3$, where the uncertainties arise from the adopted fitting procedure.

Keywords Negative muon capture · Deuteron · ${}^3\text{He}$ · Chiral effective field theory

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1 Introduction

There is a significant body of experimental and theoretical work on muon captures in light nuclei, motivated by the fact that the theoretical framework used to study these reactions is the same as that used for weak capture reactions of astrophysical interest, not accessible experimentally. Muon captures, whose rates can be measured, can therefore provide a valuable test of this theoretical framework [1].

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Very recently [2], the muon capture reactions ${}^2\text{H}(\mu^-, \nu_\mu)nn$ and ${}^3\text{He}(\mu^-, \nu_\mu){}^3\text{H}$ have been studied simultaneously in a consistent framework. In particular, the initial and final $A = 2$ and 3 nuclear wave functions have been obtained from the Argonne v_{18} (AV18) [3] or the chiral N^3LO (N^3LO) [4] two-nucleon potential, in combination with, respectively, the Urbana IX (UIX) [5] or chiral N^2LO (N^2LO) [6] three-nucleon potentials in the case of $A = 3$. The weak current consists of polar- and axial-vector components. The former are related to the isovector piece of the electromagnetic current via the conserved-vector-current (CVC) hypothesis. These and the axial current have been derived within two different frameworks, the standard nuclear physics approach (SNPA), and chiral effective field theory. The first one goes beyond the impulse approximation, by including meson-exchange current contributions and terms arising from the excitation of Δ -isobar degrees of freedom. The second approach includes two-body contributions derived in heavy-baryon chiral perturbation theory within a systematic expansion, up to N^3LO [7,8]. To be noticed that, since the transition operator matrix elements are calculated using phenomenological wave functions, it should be viewed as a hybrid chiral effective field theory approach (EFT*). Both SNPA and EFT* frameworks have been used in studies of weak pp and hep capture reactions in the energy regime relevant to astrophysics [7,9,10]. The only parameter in the SNPA nuclear weak current model is present in the axial current (the N -to- Δ axial coupling constant) and is determined by fitting the experimental Gamow-Teller matrix element in tritium β -decay (GT^{EXP}). The SNPA weak vector current, related to the isovector electromagnetic current via CVC, reproduces the trinucleon magnetic moments to better than 1 % [2]. In the case of EFT*, three low-energy constants (LECs) appear: one in the axial-vector component, and two in the electromagnetic current. Of these, only one is relevant to the weak vector current, since the other appears in front of an isoscalar operator. The corresponding coupling constants are parameters fixed to reproduce, respectively, GT^{EXP} and $A = 3$ magnetic moments. To be noticed that the EFT* currents are obtained performing the Fourier transform from momentum- to coordinate-space with a Gaussian regulator characterized by a cutoff Λ , varied between 500 and 800 MeV. The total capture rates have been found to be $392.0(2.3) \text{ s}^{-1}$ for $A = 2$ and $1484(13) \text{ s}^{-1}$ for $A = 3$. The spread accounts for the model dependence, i.e., the dependence on the input Hamiltonian model, the model for the nuclear transition operator, and, in the EFT* calculation, the cutoff sensitivity. This weak model dependence is a consequence of the procedure adopted to constrain the weak current. These results are in very good agreement with the experimental data, in particular with the very accurate measurement of Ref. [11] for the total rate in muon capture on ${}^3\text{He}$.

The muon capture on deuteron has been studied also using the SNPA and the EFT* framework in Ref. [12]. The SNPA retains two-body meson-exchange currents derived from the hard pion chiral Lagrangians of the $N\Delta\pi\rho\omega a_1$ system and are significantly different from those of Ref. [2]. On the other hand, the EFT* currents are similar to those of Ref. [2], but two differences need to be remarked: (i) the LEC appearing in the axial-vector component (d_R) is not fixed to reproduce GT^{EXP} , rather the doublet capture rate calculated in SNPA; (ii) a term is added to the leading axial two-body currents, in order to satisfy the partially-conserved-axial-current (PCAC) hypothesis, as constructed in Ref. [13] (called there, and from now on, potential current). The calculated SNPA values for the total capture rate are in the range of $416\text{--}430 \text{ s}^{-1}$, depending on the potential model used, resulting in a model dependence much larger than in Ref. [2]. It is also argued that “omitting the potential current causes an enhancement of the doublet transition rate $A_{1/2}$ by $\simeq 1\%$ ” [12].

In the present work we repeat the calculation of Ref. [2], in the EFT* approach, adding the potential currents as in Ref. [12]. We restrict our calculation to the AV18 and AV18/UIX potential models, and to a cutoff value of 600 MeV, since, as shown in Ref. [2], the dependence on these inputs is less than 1 %. We fit the d_R coefficient to GT^{EXP} , and consistently calculate the total rates for muon capture on deuteron and ^3He . The comparison with the results of Ref. [2] will give an indication of how significant are the potential current contributions for these muon captures.

The paper is organized as follows: in Sec. 2 the theoretical formalism used in the calculation is briefly reviewed. In Sec. 3 the EFT* model for the weak current is described, with the addition of the potential currents. In Sec. 4, the results are presented and discussed, and some concluding remarks are given.

2 Theoretical formalism

We briefly review the formalism used in the calculation for the muon capture processes, discussed at length in Refs. [2, 14]. The muon capture on deuteron and ^3He is induced by the weak interaction Hamiltonian [15], $H_W = \frac{G_V}{\sqrt{2}} \int d\mathbf{x} l_\sigma(\mathbf{x}) j^\sigma(\mathbf{x})$, where G_V is the Fermi coupling constant, $G_V = 1.14939 \times 10^{-5} \text{ GeV}^{-2}$ [16], and l_σ and j^σ are the leptonic and hadronic current densities, respectively. The transition amplitude can be written as

$$T_W(f, f_z; s_1, s_2, h_\nu) \equiv \langle nn, s_1, s_2; \nu, h_\nu | H_W | (\mu, d); f, f_z \rangle \\ \simeq \frac{G_V}{\sqrt{2}} \psi_{1s}^{\text{av}} \sum_{s_\mu s_d} \langle \frac{1}{2} s_\mu, \frac{1}{2} s_d | f f_z \rangle l_\sigma(h_\nu, s_\mu) \langle \Psi_{\mathbf{p}, s_1 s_2}(nn) | j^\sigma(\mathbf{q}) | \Psi_d(s_d) \rangle, \quad (1)$$

for muon capture on deuteron, \mathbf{p} being the nn relative momentum, and [14]

$$T_W(f, f_z; s'_3, h_\nu) \equiv \langle ^3\text{H}, s'_3; \nu, h_\nu | H_W | (\mu, ^3\text{He}); f, f_z \rangle \\ \simeq \frac{G_V}{\sqrt{2}} \psi_{1s}^{\text{av}} \sum_{s_\mu s_3} \langle \frac{1}{2} s_\mu, \frac{1}{2} s_3 | f f_z \rangle l_\sigma(h_\nu, s_\mu) \langle \Psi_{^3\text{H}}(s'_3) | j^\sigma(\mathbf{q}) | \Psi_{^3\text{He}}(s_3) \rangle, \quad (2)$$

for muon capture on ^3He . In order to account for the hyperfine structure in the initial system, the muon and deuteron or ^3He spins are coupled to states with total spin $f = 1/2$ or $3/2$ in the deuteron case, and $f = 0$ or 1 in the ^3He case. In Eqs. (1) and (2) we have defined with s_μ (h_ν) the muon spin (muon neutrino helicity). The Fourier transform of the nuclear weak current has been introduced as

$$j^\sigma(\mathbf{q}) = \int d\mathbf{x} e^{i\mathbf{q}\cdot\mathbf{x}} j^\sigma(\mathbf{x}) \equiv (\rho(\mathbf{q}), \mathbf{j}(\mathbf{q})) \quad , \quad (3)$$

with the leptonic momentum transfer \mathbf{q} defined as $\mathbf{q} = \mathbf{k}_\mu - \mathbf{k}_\nu \simeq -\mathbf{k}_\nu$, \mathbf{k}_μ and \mathbf{k}_ν being the muon and muon neutrino momenta. The function ψ_{1s}^{av} has been introduced to take into account the initial bound state of the muon in the atom and the charge distribution of the nucleus. It is typically approximated as [15] $|\psi_{1s}^{\text{av}}|^2 = \frac{(\alpha \mu_{\mu d})^3}{\pi}$ for muon capture on deuteron, and [14] $|\psi_{1s}^{\text{av}}|^2 = \mathcal{R} \frac{(2\alpha \mu_{\mu ^3\text{He}})^3}{\pi}$ for muon capture on ^3He , where α is the fine structure constant ($\alpha = 1/137$), $\mu_{\mu d}$ and $\mu_{\mu ^3\text{He}}$ are the reduced masses of the (μ, d) and $(\mu, ^3\text{He})$ systems, and the factor \mathcal{R} approximately accounts for

the finite extent of the nuclear charge distribution [15] and is taken to be 0.98, as in Ref. [2].

In the case of muon capture on deuteron, the final state wave function $\Psi_{\mathbf{p},s_1s_2}(nn)$ is expanded in partial waves, and the calculation is restricted to total angular momentum $J \leq 2$ and orbital angular momentum $L \leq 3$, i.e., in a spectroscopic notation, to 1S_0 , 3P_0 , 3P_1 , 3P_2 – 3F_2 and 1D_2 . Standard techniques [10,15] are now used to carry out the multipole expansion of the weak charge, $\rho(\mathbf{q})$, and current, $\mathbf{j}(\mathbf{q})$, operators. Details of the calculation can be found in Ref. [2]. Here we only note that all the contributing multipole operators selected by parity and angular momentum selection rules are included, as explained in Ref. [2].

The total capture rate for the two reactions under consideration is then defined as

$$d\Gamma = 2\pi\delta(\Delta E)\overline{|T_W|^2} \times (\text{phase space}) , \quad (4)$$

where $\delta(\Delta E)$ is the energy-conserving δ -function, and the phase space is $d\mathbf{p} d\mathbf{k}_\nu/(2\pi)^6$ for muon capture on deuteron and just $d\mathbf{k}_\nu/(2\pi)^3$ for muon capture on ^3He . The following notation has been introduced: (i) for muon capture on deuteron

$$\overline{|T_W|^2} = \frac{1}{2f+1} \sum_{s_1s_2h_\nu} \sum_{f_z} |T_W(f, f_z; s_1, s_2, h_\nu)|^2 , \quad (5)$$

and the initial hyperfine state has been fixed to be $f = 1/2$; (ii) for muon capture on ^3He

$$\overline{|T_W|^2} = \frac{1}{4} \sum_{s'_3h_\nu} \sum_{ff_z} |T_W(f, f_z; s'_3, h_\nu)|^2 , \quad (6)$$

and the factor 1/4 follows from assigning the same probability to all different hyperfine states.

After carrying out the spin sums, the differential rate for muon capture on deuteron ($d\Gamma^D/dp$) and the total rate for muon capture on ^3He (Γ_0) are easily obtained, and their expressions can be found in Ref. [2]. In order to obtain the total rate Γ^D for muon capture on deuteron, $d\Gamma^D/dp$ is plotted versus p and numerically integrated.

Bound and continuum wave functions for both two- and three-nucleon systems entering in Eqs. (1) and (2) are obtained with the hyperspherical-harmonics (HH) expansion method. This method, as implemented in the case of $A = 3$ systems, has been reviewed in considerable detail in a series of recent publications [17,18,19]. We have used the same method in the context of $A = 2$ systems, for which of course wave functions could have been obtained by direct solution of the Schrödinger equation. A detailed discussion for the $A = 2$ wave functions is given in Ref. [2].

3 The nuclear weak current operator

The chiral effective field theory weak current transition operator is taken from Refs. [7] and [8], as reviewed in Ref. [2]. It is derived in covariant perturbation theory based on the heavy-baryon formulation of chiral Lagrangians by retaining corrections up to N^3LO . The one-body operators are those listed in Eqs. (17) of Ref. [7] and (4.13)–(4.14) of Ref. [2]. The vector charge and axial current operators retain terms up to $1/m^2$, while the axial charge and vector current operators retain terms up to $1/m^3$, m being the nucleon mass. Both $1/m^2$ and $1/m^3$ contributions arise when the non-relativistic reduction of the single-nucleon covariant current is pushed to next-to-leading order. The

two-body vector currents are obtained from the two-body electromagnetic currents via CVC. These are decomposed into four terms [8]: the soft one-pion-exchange (1π) term, vertex corrections to the one-pion exchange ($1\pi C$), the two-pion exchange (2π), and a contact-term contribution. Their explicit expressions can be found in Ref. [8]. All the 1π , $1\pi C$ and 2π contributions contain low-energy constants estimated using resonance saturation arguments, and Yukawa functions obtained by performing the Fourier transform from momentum- to coordinate-space with a Gaussian regulator characterized by a cutoff Λ . Here, as discussed above, we fixed the value of $\Lambda = 600$ MeV. The contact-term electromagnetic contribution is given as sum of two terms, isoscalar and isovector, each one with a coefficient in front (g_{4S} and g_{4V}) fixed to reproduce the experimental values of triton and ^3He magnetic moments. For the AV18/UIX Hamiltonian model, with $\Lambda = 600$ MeV, $g_{4S} = 0.55(1)$ and $g_{4V} = 0.793(6)$, the error being due to numerics [2]. Note that only the isovector contribution is of interest here, but anyway it turns out to be negligible.

The two-body axial current operator consists of two contributions: a one-pion exchange term and a (non-derivative) two-nucleon contact-term. The explicit expression for the contact term can be found in Ref. [7]. Here we review the one-pion exchange term, since we add, in accordance with Ref. [13], the potential current contributions. Therefore, in momentum-space, the one-pion exchange term reads:

$$\begin{aligned} \mathbf{j}_{ij}^\pi(\mathbf{q}; A) = & \frac{g_A}{2mf_\pi^2} \left[(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)^a \left[\frac{1}{2} (1 - g_A^2) \frac{\mathbf{p}_i + \mathbf{p}'_i}{2} + \left(\frac{1}{4} + \hat{c}_4 \right) \boldsymbol{\sigma}_i \times \mathbf{k}_j \right. \right. \\ & \left. \left. + \left(\frac{1 + c_6 + g_A^2}{4} \right) \boldsymbol{\sigma}_i \times \mathbf{q} \right] + 2\hat{c}_3 \boldsymbol{\tau}_j^a \mathbf{k}_j \right. \\ & \left. - \frac{g_A^2}{4} \boldsymbol{\tau}_j^a \left(\mathbf{q} + i\boldsymbol{\sigma}_i \times (\mathbf{p}_i + \mathbf{p}'_i) \right) \right] \frac{\boldsymbol{\sigma}_j \cdot \mathbf{k}_j}{m_\pi^2 + \mathbf{k}_j^2} + i \leftrightarrow j. \end{aligned} \quad (7)$$

where $\mathbf{k}_{i,j} = \mathbf{p}'_{i,j} - \mathbf{p}_{i,j}$, with $\mathbf{p}_{i,j}$ and $\mathbf{p}'_{i,j}$ being the initial and final single nucleon momenta, $\mathbf{q} = \mathbf{k}_i + \mathbf{k}_j$, $g_A = 1.2654$ is the axial-vector coupling constant, and $f_\pi = 93$ MeV is the pion decay constant. The values used for the coupling constants \hat{c}_3 , \hat{c}_4 , and c_6 , as obtained from πN data, are $\hat{c}_3 = -3.66$, $\hat{c}_4 = 2.11$ and $c_6 = 5.83$ [7]. The terms proportional to g_A^2 in Eq. (7) are the potential currents. They are the same as in Eq. (21) of Ref. [13] or Eq. (A.17) of Ref. [12]. The low-energy constant d_R , determining the strength of the contact-term two-body axial contribution, has been fixed by reproducing GT^{EXP} , finding $d_R = 1.54(8)$. This value should be compared with the corresponding one given in Ref. [2] (see Table V), $d_R = 1.75(8)$. The difference between these two values of $\simeq 13\%$ is due to the presence of the potential currents and is comparable with that of Ref. [12].

4 Results

We present in Table 1 the results for the total rates of muon capture on deuteron, in the doublet hyperfine state (Γ^D), and on ^3He (Γ_0). The deuteron, nn , ^3He and ^3H wave functions have been calculated with the AV18 [3] two- and, when necessary, UIX [5] three-nucleon interactions. The model for the nuclear weak transition operator has been presented in Sec. 3. We compare our results with those of Ref. [2], obtained with the same Hamiltonian model and cutoff Λ , but without the two-body potential currents elaborated in Ref. [13] and discussed in Sec. 3. Note that here $d_R = 1.54(8)$,

Table 1 Total rate for muon capture on deuteron and ${}^3\text{He}$, in s^{-1} . In the $A = 2$ case, the different partial wave contributions are indicated. The numbers among parentheses indicate the theoretical uncertainties arising from the adopted fitting procedures. Such uncertainty is not indicated when less than 0.1 s^{-1} . The AV18 and AV18/UIX interactions have been used to calculate the $A = 2$ and $A = 3$ wave functions. The corresponding results of Ref. [2] are also listed.

	1S_0	3P_0	3P_1	3P_2	1D_2	3F_2	Γ^D	Γ_0
Present work	250.1(8)	20.2	46.1	71.3	4.5	0.9	393.1(8)	1488(9)
Ref. [2]	250.0(8)	19.8	46.3	71.1	4.5	0.9	392.6(8)	1488(9)

while in Ref. [2] $d_R = 1.75(8)$. From inspection of the table, we conclude that the two calculations are in remarkable agreement with each other: the differences in Γ_0 and Γ^D are $\leq 0.1 \%$, well below the theoretical uncertainties. The largest difference, of the order of 2% , is in the 3P_0 partial wave contribution to Γ^D . However, when all the partial wave contributions are summed up, the difference in Γ^D returns well below the 1% level.

In conclusion, we have studied the potential currents dictated by PCAC, as elaborated in Refs. [12,13], and we have found that their contributions to the total rates of muon capture on deuteron and ${}^3\text{He}$ are tiny. This result is a consequence of the procedure adopted to constrain the weak current. Finally, we expect that the potential currents will give tiny contributions also in weak capture reactions of astrophysical interest and in those processes whose momentum transfer is small.

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